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IMPROVED TEST FOR DETECTING EXPLOSIVE BUBBLES

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Abstract: Recent decades have witnessed a series of damages in the financial sector due to the unpleasant movements of prices beyond certain limits. These movements are commonly termed as Financial Bubbles. The formation and burst of a bubble creates huge damage in the field of finance. Hence in order to prevent the market from facing damages, the detection and modeling of financial bubble is very essential.

We proposed improved test procedures for detecting financial bubbles by combining the existing Max test and Supremum Augmented Dickey Fuller (SADF) test generally used for detecting bubbles. The performance of proposed test is compared with existing tests via Monte Carlo simulation. It is observed that the proposed test have higher power compared to the existing tests, for detecting collapsible bubble irrespective of window length and collapsible probability. Further the power of proposed test increases as window size decreases. The empirical study of S&P 500 monthly data from January 2006 to December 2010 is carried out to demonstrate the advantages of proposed test procedures over existing tests.

Keywords: ADF test, Collapsible bubble, Max test, Window length.

1. Introduction

Recent decades have witnessed a series of damages in the financial sector due to the unpleasant movements of prices beyond certain limits. These changes in the price of an asset without any intuitive reasons will lead to market imbalance. For different reasons, a speculator trusts that the demand for a stock will continue to rise or that the stock will become profitable within a short period. For example, a situation in which the price of a stock rises far above from its actual value may motivate the investor to invest on it. This craze continues until the investor realizes that there is no profit in investment. Then onwards the market value of the asset which was hiked to an unjustified level starts to fall which forms a bubble structure in the time series. This type of structure in the financial market is commonly known as financial bubble.

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A Financial bubble can be generally defined as a transient upward acceleration of prices above fundamental value. Kindleberger defines a bubble as ‘a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers – generally the speculators’.

The mathematical definition of a bubble can be given using asset pricing model of Lucas (1978).

$$P_t = \frac{1}{(1+r)} E_t(P_{t+1} + d_{t+1}) \qquad (1)$$

where E_t denotes conditional expectations on information at time t, P_t is asset price at time t, d_t is the payoff received from asset price and r is net interest rate. Forward iteration of this first degree difference equation lead to final solution as

$$F_t = \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} E_t d_{t+1} \qquad (2)$$

In the literature, this forward looking solution is referred as market fundamentals or simply fundamental price of asset.

The solution of equation (1) is unique i.e P_t=F_t, if the following transversality condition

$$\lim_{k\to\infty} E_t\left[\frac{1}{(1+r)^k} P_{t+k}\right] = 0 \qquad (3)$$

is satisfied. Otherwise there will be infinite number of solutions. The general form of the solution is given by,

$$P_t = F_t + B_t \qquad (4)$$

where P_t is asset price, F_t is fundamental price and B_t represents excess price.

Initially, Shiller (1981) and LeRoy & Porter (1981) developed variance bound tests to explain fluctuations in the price of a stock. Later the works of Tirol (1982) and Blanchard &Watson (1982) revealed that violation of the variance bounds could be considered as a test for the presence of bubbles in the series. But this procedure was highly criticized by a number of researchers like Flavin (1983), Marsh and Merton (1986), Cox (2011) and many more. Cochrane (1992) provided an example of violation of variance bound due to time varying discount rate rather than a bubble. So it is suggested not to use tests based on variance bound to detect bubbles.

In a different approach, West (1987) developed a two-step test for identifying bubbles based on Euler’s equation of no arbitrage process and the autoregressive process of dividends. This test tries to tackle two problems “model misspecifications” and “bubbles” simultaneously by testing the above problems in sequence. However, Flood & Hodrick (1986) and Dezbakhsh & Demirguc-Kunt (1990) criticized this test procedure because it exhibited significant size distortions in small samples.

In the financial literature, it is well known that if bubbles are present in the market, they should possess explosive behavior in the asset price (P_t) so that stationarity cannot be attained even after taking multiple differences. This feature motivated Diba and Grossman (1988) to test for the presence of bubbles by applying unit root test to P_t. Further they proved that P_t and d_t cannot co move or co integrate if d_t itself is not explosive. Thus if the discount rate is independent of time, then the test for explosive behavior in the price series is equivalent to the test for financial bubble.

However Evans (1991) demonstrated that the unit root and co integration tests are unable to detect a class of bubbles that are always positive but periodically collapse. It is noted that such type of bubbles behave much like an I(1) process or even like a stationary linear auto regressive process provided that the probability of collapse of bubble is not negligible. The periodically collapsible bubbles can be represented by following data generating process.

$$B_{t+1} = \begin{cases} \rho^{-1} B_t u_{t-1} & if \quad B_t \le \alpha \\ [\xi + (\lambda\rho)^{-1}\theta_{t+1}(B_t - \rho\xi)]u_{t+1} & if \quad B_t > \alpha \end{cases}$$

where $\mu_t \sim \exp(y_t - \tau^2/2)$; $y_t \sim N(0, \tau^2)$ and θ_t is a exogenous independently and identically distributed Bernoulli process which takes value one with probability λ and zero with probability (1-λ).

There for, there is a need for a powerful test to detect periodically collapsible bubbles in the price series. In the recent decades many test procedures to detect such type of bubbles based on “unit root” in the price series were proposed. In this paper we proposed test procedure which has better power to detect collapsible bubbles than the existing tests.

In the next section we present the details of the unit root test procedure and some of its modified versions especially developed to detect financial bubbles. Section 3 gives the details of proposed test procedures. Comparison of the proposed test procedures with the existing tests in terms of power is presented in the Section 4. The working of the proposed tests is illustrated with an example in Section 5 and conclusion is presented in the last Section.

2. Materials and Methodology

2.1 Augmented Dickey Fuller (ADF) Test:

Fuller and Dickey & Fuller (1979, 1981) proposed a test procedure to detect unit root in the time series based on following three versions of regression equations.

i) Unit root without drift and deterministic time trend

$$\Delta y_t = \beta y_{t-1} + \sum_{i=1}^{k} \varphi_i \Delta y_{t-i} + \varepsilon_t \,, \varepsilon_t \sim N(0, \sigma^2) \qquad (6)$$

ii) Unit root with drift

$$\Delta y_t = a + \beta y_{t-1} + \sum_{i=1}^{k} \varphi_i \Delta y_{t-i} + \varepsilon_t , \varepsilon_t \sim N(0,\sigma^2) \qquad (7)$$

iii) Unit root with drift and deterministic time trend

$$\Delta y_t = a + b_t + \beta y_{t-1} + \sum_{i=1}^{k} \varphi_i \Delta y_{t-i} + \varepsilon_t , \varepsilon_t \sim N(0,\sigma^2) \qquad (8)$$

For some given number of lag k. Depending upon the model version (6), (7) and (8), the test statistic for testing the null hypothesis H_0: β=0 against H_1: β>0 follows a non standard distribution. Hence one has to determine the critical values for this test by simulation. In the literature this statistic is commonly known as ADF statistic.

2.2 Max Test

Leybourne (1995) proposed a new test procedure by taking the maximum of the ADF statistic (data in forward direction) and ADF statistic (data in reversed direction). Formally, the test statistic for Max test can be denoted as follows.

$$Max = \max\left(ADF^{f}, ADF^{r}\right) \qquad (9)$$

where ADF^f denotes the ADF statistic computed for the data in forward direction and ADF^r denotes ADF statistic for the data in reverse order. He showed that this test performs better compared to conventional ADF test to detect collapsible bubble via Monte Carlo simulation.

2.3 Supremum ADF Tests (SADF Tests)

Phillips et al. (2011) observed that the periodically collapsing behavior of the bubbles can be captured by applying the ADF test procedure on the sub samples. They proposed two SADF test procedures namely forward recursive SADF test and forward rolling window SADF test. Further they proved that these test procedures are capable of differentiating between periodically collapsing bubbles and unit root processes. The procedure for both tests is essentially same. But they differ in the selection criteria of subsamples.

a) Forward recursive SADF Test

In this procedure, first compute a sequence of ADF statistic on the subsamples which expands by one at each pass. Then find the supremum ADF statistic of this sequence. Formally one can represent the test statistic as follows.

$$\underset{r_w \in [r_0,1]}{SADF} = \sup_{r_w \in [r_0,1]} \left(ADF_{r_w}^{f} \right) \qquad (10)$$

where r_0 is the size of initial subsample. Note that there is no hard rule to select r_0. A part of sample say, $[r_0]$ where [.] denotes the greatest integer part of r_0 is selected as the size of initial subsample. Usually r_0 lies between 0.1 and 0.3. i.e. selecting initial 10% to 30% of observations from the original sample as the size of first subsample.

b) Forward rolling SADF Test

Here, first determine a series of ADF statistic on the subsamples which rolls ahead with constant sample size so that the starting and ending points of each subsample is incremented by one at each pass. The test statistic for forward rolling SADF test can be represented as follows.

$$\underset{r_w}{SADF} = \sup_{r_w} \left(ADF_t^{f} \right); t = 1,2,3,... \qquad (11)$$

where r_w denotes the size of rolling window.

To select r_w, Phillips et al. (2015) suggested a method based on level of significance (α) and sample size (n). They argued that r_w needs to be chosen according to the total number of sample observations n. If n is small, r_w need to be large enough to ensure that there are enough observations for adequate initial estimation. If n is large, r_w can be set to a smaller number so that the test does not miss any opportunity to detect an early explosive episode. Thus for practical usage they have recommended a formula to determine r_w which is given below.

$$r_w = \alpha + \frac{1.8}{\sqrt{n}} \qquad (12)$$

Based on the sample observations only, a new procedure to determine r_w is introduced in this paper. In this procedure first decide a large interval for window say 0.1 to 0.3. Then obtain forward rolling window SADF statistic for each window in the interval (γ_w=0.10, 0.11, 0.12…0.30). Then fix $r_w = \gamma_w$ corresponding to the maximum value for ADF statistic in the sequence.

3. Improved Test for Detecting Collapsible Bubble

From the above discussions, it is clear that Max test improves the power of ADF test and SADF tests were more suitable for detecting collapsible bubble. Thus in this paper we proposed a set of improved tests namely, Max recursive SADF test and Max rolling window SADF test by combining them. The rationality to do so is to take the advantage of Max test and SADF test in a single test procedure so that one can detect bubbles in the time series with greater efficiency.

3.1 Max SADF Tests (MSADF Tests)

As the name itself suggest, here first we compute the SADF test statistic for the observations in the forward and reverse direction separately. Then the maximum among the set of statistic computed is treated as MSADF test statistic. The test statistic for two test procedures can be represented as follows.

$$\underset{r_w \in [r_0,1]}{MSADF} = \sup_{r_w \in [r_0,1]} \left(ADF_{r_w}^{f}, ADF_{r_w}^{r} \right) \tag{13}$$

$$\underset{r_w}{MSADF} = \sup_{r_w} \left(ADF_t^{f}, ADF_t^{r} \right); t = 1,2,3,... \tag{14}$$

where r_0 is the size of initial subsample and r_w is the size of rolling window which is determined as explained earlier.

4. Power Comparison

Now we compare the powers of conventional, SADF and newly proposed Max SADF test procedures via 5000 Monte Carlo simulation. Throughout the analysis, ADF regression model with drift is considered.

$$\Delta y_t = a + \beta y_{t-1} + \sum_{i=1}^{k} \varphi_i \Delta y_{t-i} + \varepsilon_t , \varepsilon_t \sim N(0, \sigma^2)$$

The optimum lag length is found to be zero which is determined using top-down sequential test procedure proposed by Campbell & Perron (1991). The suitable window length for SADF and Max SADF tests were selected as explained above. Note that w1, w2 and w3 represent the size of the recursive window with 10%, 20% and 30% of sample observations respectively. w4 represent the size of rolling window selected using the method suggested by Phillips and w5 represent the size of rolling window selected using new procedure introduced in this paper. To compute critical values, the observations were simulated using simple random walk model. Table 1 and Table 2 reports the critical values for the different test procedures under study.

Table 1: Represents the critical values at 5% significant level for conventional ADF test.

Method	Critical value
Conventional ADF	-0.0893
Max conventional ADF	0.2598

Note: The right tailed critical values are obtained via 5000 Monte Carlo simulation with sample size =200.

Table 2: Represents the critical values at 5% significant level for SADF test.

	Recursive			Rolling	
	w_1=0.1	w_2=0.2	w_3=0.3	w_4	w_5
SADF	1.4404	1.3059	1.2139	1.7123	2.0469
Max SADF	1.6922	1.5407	1.4341	1.9263	2.2622

Note: The right tailed critical values are obtained via 5000 Monte Carlo simulation with sample size =200. Here w_1=20, w_2=40, w_3=60, w_4=35 and w_5=20.

The price series with collapsible bubble for simulation study is generated using (4) where

$$B_{t+1} = \begin{cases} \rho^{-1} B_t u_{t-1} & if \quad B_t \le \alpha \\ [\xi + (\lambda\rho)^{-1}\theta_{t+1}(B_t - \rho\xi)]u_{t+1} & if \quad B_t > \alpha \end{cases}$$

and

$$F_t = \frac{1-g}{g} + \frac{1-g}{(1+g)g^2}u + \frac{r_t}{g} \qquad where \qquad r_t = c + r_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0,\sigma^2)$$

The values of the parameters were set to B_0=0.5, α=1, ξ=B_0, u=0.373, σ=0.1574, g=0.05, τ=0.0025 and sample size n=200 as in Evans (1991) in order to facilitate comparison. Table 3 represents the power of conventional ADF and Max conventional ADF test procedures. The power of recursive SADF and rolling window SADF test procedures were reported in Table 4 and Table 5 respectively.

Table 3: Represents the power of conventional and Max conventional ADF test.

	Collapsible Probability			
	λ=0.90	λ=0.50	λ=0.20	λ=0.10
Conventional ADF	0.4294	0.4832	0.4930	0.4972
Max conventional ADF	0.5746	0.6784	0.7014	0.7072

Note: The power is computed via 5000 Monte Carlo simulation with sample size =200. The test is carried out based on the hypothesis H_0: β=0 against H_1: β>0.

Table 4: Represents the power of Forward recursive and Max recursive SADF test.

Recursive window(w_i)	**Collapsible Probability(λ)**				
	Method	λ=0.90	λ=0.50	λ=0.20	λ=0.10
w_1=0.1	Forward recursive SADF	0.6896	0.5838	0.5466	0.5406
	Max recursive SADF	0.7136	0.6410	0.6192	0.6168
w_2=0.2	Forward recursive SADF	0.6666	0.5320	0.5152	0.5150
	Max recursive SADF	0.7006	0.6012	0.5976	0.6016
w_3=0.3	Forward recursive SADF	0.6002	0.4938	0.4856	0.4888
	Max recursive SADF	0.6620	0.5846	0.5866	0.5866

Note: The power is computed via 5000 Monte Carlo simulation with sample size =200. The test is carried out to test the hypothesis H_0: β=0 against H_1: β>0 by taking window lengths w_1=20, w_2=40 and w_3=60.

Table 5: Represents the power of Forward rolling and Max rolling SADF test

Rolling window(w_i)	**Collapsible Probability(λ)**				
	Method	λ=0.90	λ=0.50	λ=0.20	λ=0.10
w_4	Forward rolling SADF	0.9222	0.8830	0.8790	0.8676
	Max rolling SADF	0.9338	0.9028	0.9012	0.8964
w_5	Forward rolling SADF	0.9028	0.8714	0.8452	0.8386
	Max rolling SADF	0.9178	0.8990	0.8814	0.8752

Note: The power is computed via 5000 Monte Carlo simulation with sample size =200. The test is carried out to test the hypothesis H_0: β=0 against H_1: β>0 by taking window lengths w_4=35 and w_5=20.

It can be clearly observed that improved test procedures have higher power compared to the existing tests for detecting collapsible bubble irrespective of window length and collapsible probability (Table 3, 4 and 5). Further, the power of proposed Max test increases as the window size decreases. For example, the power of recursive test is higher in case of window length 0.1 than with 0.3. However, one should note that the power of SADF and Max SADF test procedures were sensitive to the selection of window length. The new proposed procedure to select rolling window length does not yield any improvement in power of the test. But it closely resembles with the method proposed by Phillips et al. (2011) and it depends only on sample observations.

5. Empirical Analysis

Many economists consider the financial crisis of 2007–08 as the worst financial crisis after the Great Depression of the 1930s. So this study uses monthly data of S&P 500 stock prices and dividends for the period from January 2006 to December 2010 obtained from Robert Shiller's webpage. The earlier studies on this series supported the presence of bubble (Phillips et al., 2015). The log price series & log dividend series were computed and summary statistics are presented in the Table 6. Figure 1 plot the log price series and log dividend series.

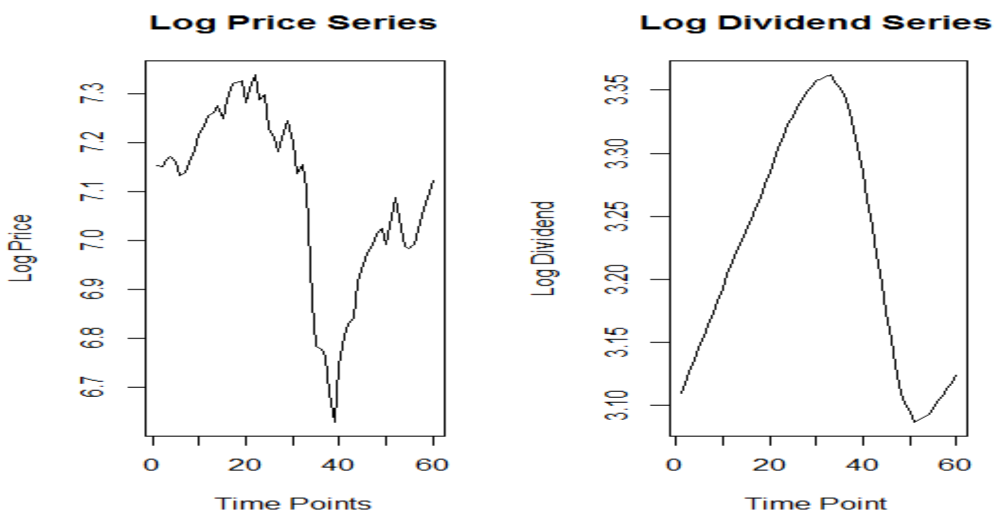

Figure1: Represent the plot of log price series and log dividend series from January-2006 to December-2010

Table 6: Represents summary statistic of S & P 500

Data	**Monthly stock price and dividend from January 2006 to December 2010**	
	Log Price Series	Log Dividend Series
Maximum	3.187425	1.460146
Minimum	2.879170	1.340642
Mean	3.079308	1.399317
Skewness	0.7077	-0.0511

Kurtosis	5.6394	4.5226

In order to compute the SADF and Max SADF tests, the optimum lag length is determined using top-down sequential test procedure and is equal to 2 and 6 for price and dividend series respectively. The initial window size for recursive ADF tests and the size of rolling window are chosen to be 18 and 28 by following the procedures explained in Section 2.3.Table 7, 8 and 9 summarizes the results of the different tests.

Table7: Represents the results of Conventional and Max conventional test.

Panel A (for price series)		
Method	Statistic	p-value
Conventional ADF	-1.328	0.401
Max conventional ADF	-1.2632	0.6068
Panel B (for dividend series)		
Conventional ADF	-1.6496	0.5696
Max conventional ADF	-1.1145	0.7858

Note: The right tailed p-values are obtained via 5000 Monte Carlo simulation. Sample size =60 are generated from simple random walk model.

Table 8: Represents the results of Forward recursive and Max recursive SADF tests.

Panel A (for price series)		
Method	Statistic	p-value
Forward recursive SADF	1.5728	0.0796
Max recursive SADF	1.4921	0.0448
Panel B (for dividend series)		
Forward recursive SADF	0.5837	0.603
Max recursive SADF	-0.3612	0.8136

Note: The right tailed p-values are obtained via 5000 Monte Carlo simulation. Sample observations =60 are generated from simple random walk model. The size of rolling window is chosen to be 1

Table 9: Represents the results of Forward rolling and Max rolling SADF tests.

Method	Statistic	p-value
Panel A (for price series)		
Forward rolling SADF	1.3405	0.0498
Max rolling SADF	1.8218	0.0360
Panel B (for dividend series)		
Forward rolling SADF	-0.3603	0.6904
Max rolling SADF	-0.6103	0.8782

Note: The right tailed p-values are obtained via 5000 Monte Carlo simulation. Sample observations =60 are generated from simple random walk model. The initial size of window is chosen to be 28.

The conventional ADF and Max conventional ADF tests do not reject the null hypothesis of "no bubble" in the price series (with p-value is 0.401 and 0.6068 respectively) and dividend series (with p-value 0.5696 and 0.7858 respectively). This may be due to the fact that conventional ADF test fails to capture "explosive" behaviour in the price series.

The analysis based on Forward rolling SADF test and Max rolling SADF test supports the explosive behaviour of price series (with p-value 0.0498 and 0.0360 respectively indicating the rejection of the null hypothesis of 'no bubble') and non explosive behaviour of dividend series (with p-value 0.6904 and 0.8782). But observe that p value corresponding to proposed Max rolling test is less than that for existing Forward rolling SADF test. This indicates that the proposed test reject the null hypothesis of 'no bubble' with better significance level.

Further the analysis based on the Max recursive SADF test justifies the presence of bubble in the time series (with p-value 0.0448). However the Forward recursive SADF test fails to detect the presence of bubble (p-value 0.0796) at 5% level of significance. This clearly indicates the advantages of proposed test procedure over the existing one.

6. Conclusion

In this paper an improved test procedure to detect collapsible bubble based on SADF test and Max test is proposed. The performance of Max recursive and Max rolling SADF tests is compared with the existing tests via 5000 Monte Carlo simulation. It is observed that the improved test procedures have higher power compared to the existing tests for detecting collapsible bubble irrespective of window length and collapsible probability. However, one should note that the power of SADF and Max SADF test procedures were sensitive to the selection of window length. The empirical study of S & P 500 monthly data from January 2006 to December 2010 based on Forward rolling SADF test and Max rolling SADF test supports the presence of bubble. But observe that p value corresponding to proposed Max rolling test is less than that for existing Forward rolling SADF test. This indicates that the proposed test reject the null hypothesis of ‘no bubble’ with better significance level. Also Max recursive SADF test supports the presence of bubble whereas Forward recursive SADF test fails to detect bubble even if the series has bubble. Therefore the proposed Max SADF test procedure has advantage over SADF test for detecting financial bubble.

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