
$$\frac{\dots}{\dots} \times \dots$$

$$\dots$$

$$\left. \dots \right\} \dots$$

$$\dots$$

$$\dots$$

$$\dots$$

$$\dots$$

and

$$\begin{aligned}\hat{f}_{d(i)} &= S(\mathbf{y}^* - \mathbf{X}b_{d(i)}) \\ &= \hat{f}_d - S[I - \mathbf{X}(\tilde{\mathbf{X}}'\tilde{\mathbf{X}} + I_p)^{-1}(\tilde{\mathbf{X}}'\tilde{\mathbf{X}} + dI_p)(\tilde{\mathbf{X}}'\tilde{\mathbf{X}})^{-1}\tilde{\mathbf{X}}'(I - S)]\zeta_i(\mathbf{y}_i - \mathbf{y}_i^*).\end{aligned}$$

Also we can verify that

$$\begin{aligned}\mathbf{y}_i - \mathbf{y}_i^* &= \zeta_i(\mathbf{y} - \mathbf{X}b_{d(i)} - \hat{f}_{d(i)}) \\ &= \zeta_i'(\mathbf{y} - H_d\mathbf{y}^*) = \zeta_i'[\mathbf{y} - H_d\mathbf{y} + H_d(\mathbf{y} - \mathbf{y}^*)] \\ &= \zeta_i'(\mathbf{y} - H_d\mathbf{y}) + \zeta_i'H_d\zeta_i(\mathbf{y}_i - \mathbf{y}_i^*),\end{aligned}$$

solving for $\mathbf{y}_i - \mathbf{y}_i^*$ we have

$$\mathbf{y}_i - \mathbf{y}_i^* = \mathbf{e}_{i,d} / (1 - h_{ii,d}),$$

from which Theorem 3.1 follows. If the non-parametric component f is not present in the model, equation (1) reduces to the the updating formula of Cook distance for ordinary ridge regression (see Walker and Brich (1988)).

3.3. Measuring Influence in LEs

Some version of Cook's distance for the component of semiparametric regression models is suggested by Eubank (1985) and Kim et al. (2001, 2002). An analogous definition of Cook's distance can be given for LEs in semiparametric regression model (1) as follows:

3.3.1. Influence on b_d

At least two versions of Cook's distances can be constructed for LE of β , namely,

$$D_{bi} = \frac{(b_d - b_{d(i)})' \mathbf{X}' \mathbf{X} (b_d - b_{d(i)})}{p_1 s^2} \quad (10)$$

and

$$D_{bi}^* = \frac{(b_d - b_{d(i)})' K [\tilde{\mathbf{X}}'(I - S)(I - S)' \tilde{\mathbf{X}}]^{-1} K (b_d - b_{d(i)})}{p_1 s^2}, \quad (11)$$

in which $p_1 = \sum_{i=1}^n \sum_{j=1}^n \tilde{h}_{ij}^2$, where \tilde{h}_{ij} is ij th element of matrix \tilde{H}_d and $K = (\tilde{\mathbf{X}}'\tilde{\mathbf{X}} + I)^{-1}(\tilde{\mathbf{X}}'\tilde{\mathbf{X}} + dI)$, D_{bi}^* is direct generalization of Cook's distance, and D_{bi}^* is based on the fact that $\text{var}(\hat{\beta}_k) = K^{-1}(\tilde{\mathbf{X}}'\tilde{\mathbf{X}})K^{-1}$.

The mean squared error is a function of the fitted values and the responses, neither of which depends on individual eigenvalues of $\tilde{\mathbf{X}}'\tilde{\mathbf{X}}$. Therefore, it is not affected by collinearity. For this reason, the regular estimators of σ^2 [s and $s_{(i)}$] will be used as measures of scale. It would be desirable to be able to write these

measures as functions of leverage and residual. By the theorem we can express D_{bi} and D_{bi}^* as a function of i th residual and leverage as follows:

$$D_{bi} = \frac{[\sum_{j=1}^n \tilde{h}_{ij}^2] e_{di}^2}{p_1 s^2 (1 - h_{d,ii})^2},$$

$$D_{bi}^* = \frac{h_{ii}^0 e_{di}^2}{p_1 s^2 (1 - h_{d,ii})^2},$$

in which h_{ij}^0 is i th diagonal elements of matrix $(I - S)\tilde{X}'(\tilde{X}'\tilde{X})^{-1}[\tilde{X}'(I - S)(I - S)'\tilde{X}]^{-1}(\tilde{X}'\tilde{X})^{-1}\tilde{X}'(I - S)$.

3.3.2. Influence on \hat{f}_d

An influence measure for i th observation on \hat{f}_d may be defined as a type of Cook's distance by

$$D_{fi} = \frac{(\hat{f}_{d(i)} - \hat{f}_d)'(\hat{f}_{d(i)} - \hat{f}_d)}{p_2 s^2}. \quad (12)$$

D_{fi} can be expressed in the form of residuals and leverage's as

$$D_{fi} = \frac{[\sum_{j=1}^n h_{ij}^{*2}] e_{d,i}^2}{p_2 s^2 (1 - h_{d,ii})^2},$$

where $p_2 = \sum_{i=1}^n \sum_{j=1}^n h_{ij}^{*2}$ and h_{ij}^* is ij th element of matrix H_d^* .

3.3.3. Influence Measure for \hat{y}_d

Among the most popular single-case influence measures is the difference in fit standardized DFFITS (Belsley et al. (1989)), which can be defined for model (1) as

$$DFFITS_d = \frac{x_i'(b_d - b_{d(i)}) + (\hat{f}_d - \hat{f}_{d(i)})}{se(\hat{y}_d)}, \quad (13)$$

the denominator is an estimator of the standard error of the LEs fitted value. It can be shown that $se(\hat{y}_d) = s[\sum_{j=1}^n h_{d,ij}^2]$ is an estimator of the standard error of the fitted values. Based on the relations in (7) and (8), the relation in (13) can be written as

$$DFFITS_d = \left[\frac{h_{d,ii}}{1 - h_{d,ii}} \right] \frac{e_{d,i}}{s[\sum_{j=1}^n h_{d,ij}^2]}.$$

An influence measure for the i th observation on the vector of fitted values can be similarly defined by

$$D_{yi} = \frac{(\hat{y}_d - \hat{y}_{d(i)})'(\hat{y}_d - \hat{y}_{d(i)})}{p_3 s^2 (1 - h_{d,ii})^2}, \quad (14)$$

we may express it as

$$D_{yi} = \frac{\sum_{j=1}^n h_{d,ij}^2 e_{d,i}^2}{p_3 s^2 (1 - h_{d,ii})^2},$$

where $p_3 = \sum_{i=1}^n \sum_{j=1}^n h_{d,ij}^2$.

3.4. Relationship between the Cook's distances

Here we shall investigate the relationships between D_{yi} , D_{bi} and D_{fi} . We have

$$\begin{aligned} p_3 s^2 D_{yi} &= (\hat{y}_d - \hat{y}_{d(i)})'(\hat{y}_d - \hat{y}_{d(i)}) \\ &= \{X(b_d - b_{d(i)}) + (\hat{f}_d - \hat{f}_{d(i)})\}' \{X(b_d - b_{d(i)}) + (\hat{f}_d - \hat{f}_{d(i)})\} \\ &= (b_d - b_{d(i)})' X' X (b_d - b_{d(i)}) + (\hat{f}_d - \hat{f}_{d(i)})' (\hat{f}_d - \hat{f}_{d(i)}) \\ &\quad + 2(b_d - b_{d(i)})' X' (\hat{f}_d - \hat{f}_{d(i)}) \\ &= p_1 s^2 D_{bi} + p_2 s^2 D_{fi} + 2(b_d - b_{d(i)})' X' (\hat{f}_d - \hat{f}_{d(i)}) \end{aligned}$$

In the presence of cross product term $2(b_d - b_{d(i)})' X' (\hat{f}_d - \hat{f}_{d(i)})$, D_{yi} might not be large even though both D_{bi} and D_{fi} are large. Conversely D_{yi} could be large even though both D_{bi} and D_{fi} are small. This is reason we should compute the influence of an observation on each estimator separately (see Kim et al. (2001, 2002)).

3.4.1. Reference values

In non-parametric or partial linear regression models, a general guideline for reference values for the Cook's distances is out of reach. After finding proper tuning parameter d , we suggest take the data-specific method based on a bootstrap idea proposed by Kim and Storer (1996). We take this bootstrap idea as follows:

1. Fit the observed data to model (1),
2. Generate n random number $\epsilon_i^* \sim N(0, s^2)$,
3. With known d , generate n pseudo-response $\hat{y}_i^* = \mathbf{x}_i' \hat{b}_d + \hat{f}_d(t_i) + \epsilon_i^*$
4. Based on $(\mathbf{x}_i, t_i, \hat{y}_i^*)$, compute n numbers of the Cook's distances defined in relations (10)-(13).

5. Repeat steps of 1-4 r times, and
6. Obtain the average of r maximum values.

3.5. Selecting the Smoothing Parameter

Both local-polynomial regression and smoothing splines have an adjustable smoothing parameter, say λ . This parameter may be selected by visual trial and error, picking a value that balances smoothness against fidelity to the data. More formal methods of selecting smoothing parameters typically try to minimize the mean-squared error of the fit or by some form of cross-validation. Here, the cross-validation or the generalized cross-validation, which might be used to estimate smoothing parameter, can be written as a function of the residuals and LEs leverages by

$$CV(\lambda) = \sum_{i=1}^n \{e_{i,d}/1 - h_{ii,d}\}^2,$$

$$GCV(\lambda) = \sum_{i=1}^n \{e_{i,d}/1 - \frac{1}{n} \text{tr}(H_d)\}^2.$$

4. Numerical Example

4.1. Simulation study

In this section, we will discuss the simulation study to investigate the effect of influence observations in multicollinear data sets. Adopting the model (1.1) we simulate the response from the following model:

$$y = X\beta + f(t) + \epsilon$$

where $X \sim N_{10}(0, \Sigma)$ and the coefficient vector β is the normalized eigenvector corresponding to the largest eigenvalue of $\tilde{X}'\tilde{X}$. We selected a smooth function of the form $f(t) = m_n f_0(t)$ where $f_0 = 4.26 [\exp(-3.25t) - 4\exp(-6.5t) + 3\exp(-9.7t)]$ with $\max |f(t)| = 9$ (i.e $m_n = 9$). The vector t and ϵ generated from $U(0, 1)$ and $N(0, 0.05)$, respectively. Three different set of data corresponding to $\Sigma = \Sigma_1$, $\Sigma = \Sigma_2$ and $\Sigma = \Sigma_3$ are considered to show the weakly, strong and severely collinear between the explanatory variables, respectively. For this, the off-diagonal elements of covariance matrix of Σ_1, Σ_2 and Σ_3 are set to be $\text{cov}(x_i, x_j) = 0.7\sigma_{x_i}\sigma_{x_j}$, $\text{cov}(x_i, x_j) = 0.9\sigma_{x_i}\sigma_{x_j}$ and $\text{cov}(x_i, x_j) = 0.99\sigma_{x_i}\sigma_{x_j}$, $i, j = 1, \dots, 10$, respectively, where $\sigma_{x_i} = 4i/5$. For each data set, 5th, 10th and 15th observations are replaced as influential observations. With given Σ_1, Σ_2 and Σ_3 the samples of 15, 50 and 100 units are produced for 5000 times. D_{yi} , D_{bi} and D_{fi} are calculated for each

data set. Table 1 shows the mean and standard error (in parenthesis) of the values of D_{yi} , D_{bi} and D_{fi} . It can be seen that at least 2 influence measures clearly identify the 5th, 10th and 15th observations as influential observations. The percentage of the correct detection of the influential observations by influence measures are given in Table 2.

4.2. Real data

The Longley (1967) data is a data frame with 7 economical variables, x_1 = GNP implicit price deflator, x_2 =Gross National Product, x_3 = number of people in the armed forces, x_4 =number of unemployed, x_5 =Population, x_6 = Year and y =number of people employed. This data has been used to explain the effect of extreme multicollinearity on the ordinary least square estimation. The scaled condition number (see Belsley et al. (1989), p. 100) of this data set is 43,275. This large value suggests the presence of an unusually high level of collinearity. Cook (1977) applied Cook's distance to this data and found that cases 5, 16, 4, 10, and 15 (in this order) were the most influential observations in OLS. Walker and Brich (1988) analysed the same data to find anomalous observations in ridge regression using the method of case deletion influential measures. They found that cases 16, 10, 4, 15 and 5 (in this order) were the most influential observations in Cook's and *DFFITs* measures. Shi and Wang (1999) detected the five same cases as Walker and Brich (1988) did but with another order, i.e., 10, 4, 15, 16, and 5, using local influence method. Recently Asar and Erisoglu (2016) and Jahufer and Chen (2011) used this data set and applied case deletion formula and local influence respectively on LE in ordinary linear regression. They found the five case i.e., 4, 6, 15 and 16 and found that the order of influence is changed respect to different d . In leverage and residual measures Asar and Erisoglu (2016) found the same influential observations but the order is changed. In this paper we used the same data set to assess the influential observations in LEs by using the method of case deletion influential measures such as Cook's distances, *DFFITs* and Leverages. For simplicity of the numerical study we consider the following semiparametric model

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + f(x_{i6}) + \epsilon_i, \quad i = 1, 2, \dots, 16$$

The smoothing spline was used and the tuning parameter $\lambda = 0.006$ was selected by minimizing the cross-validation criterion. The LE biasing parameter is estimated for this data set with $d = 0.962$. Table 1 show the three type Cook's distances, D_{bi} , D_{gi} and D_{yi} for $d = 0$ and $d = 0.962$ model respectively. The reference values are evaluated based on the algorithm described in Section 4 with $r = 100$. It can be seen that cases 5, 6, 10, and 15 in this order are most influential points on d and \hat{f}_d . In both influential measures the detected influential cases exactly same but only, the influence of observation 15 on \hat{f}_d is not significant.

However, the influence observation on \hat{y}_d are changed in such a way cases 5, 16, 4 and 10 (in this order) are the most significant influential observations on \hat{y}_d . We note one should be careful in comparing the three Cooks distances D_{bi} , D_{fi} and D_{yi} . Since different normalizing constants p_1 , p_2 and p_3 are used, direct comparison may be misleading. For example, when $D_{f5}=0.135$ and $D_{y5}=0.561$, it is not reasonable to conclude that the 5th observation is 4 times more influential on \hat{y}_d than on \hat{f}_d . Each Cooks distance should be used and interpreted within the estimator. For example, when $D_{f10}=0.462$ and $D_{f15}=0.191$, we may conclude that the 10th observation is three times more influential than the 15th observation on \hat{f}_d .

We have also plotted the most influential cases, discovered using (10), (12) and (13), to study the impact of d on these cases against various values of d in Fig.1. The impact of d on the influence of each case is clear. The influential cases are similar in Fig.1(a) and Fig.1 (b) but the order of influence magnitude varies. Specifically, we observe that although case 5 is the most influential, with reference to both (10) and (12), its influence decreases sharply as d increases. In contrast, the influence of case 6 increases slowly as d increases. There is little difference in Fig. (c) the influential observations are 5, 16, 4 and 10 (in this order) and the order of influence along d is unchanged. The impact of d for influence observation on \hat{y}_d is different from d and \hat{f}_d . It is seen that the influence of cases 5 and 4 slowly decreases while influence of case 6 is slowly increases as d grows. The influence of the case 10 remains relatively constant in Fig.(a)-(c). DFFITS, leverages ($h_{d,i}$) and residuals are plotted against d in Fig.(3). The influential observations are the same as those in the Cook's measures, but the order of magnitude is changed. In DFFITS and leverage plot, as the plots shows the DFFITS and leverage of case 6 increases as d grows, whereas case 16 and 5 remains as the highest point. The residual, on the other hand is more constant for all cases relative to other plot. The joint effect of leverage and residuals is reflected in the change of influence observed in Fig.(1)(c). Fig (3). contain the plot of \hat{f}_d when the influential observations are deleted and when $d = 0.962$.

5. Summary

In this article, we discussed case omission measures for semiparametric models with the LE estimators. In general linear model Belsley et al. (1989) noted that biased estimators are used to reduce the affect of multicollinearity and that the influence of some cases can be modified. Based on this fact, they suggested that multicollinearity should be controlled before attempting to measure influence. In this paper, we show that, in semiparametric regression models when LE is used, the influence of some case can be changed along d . We found that when the value of d is determined, influence measures should be computed for that d . The main advantage of deletion formulas derived in theorem 1 is that, the

estimator does not have to be computed every time a case is deleted. For a value of d all of the elements in (10)-(13) are readily available from a single run of LE. Also, these measures, based on deletion formulas, are particularly helpful for large data sets. Furthermore, the deletion formulas provide computationally inexpensive influence measures for LE in semiparametric regression models.

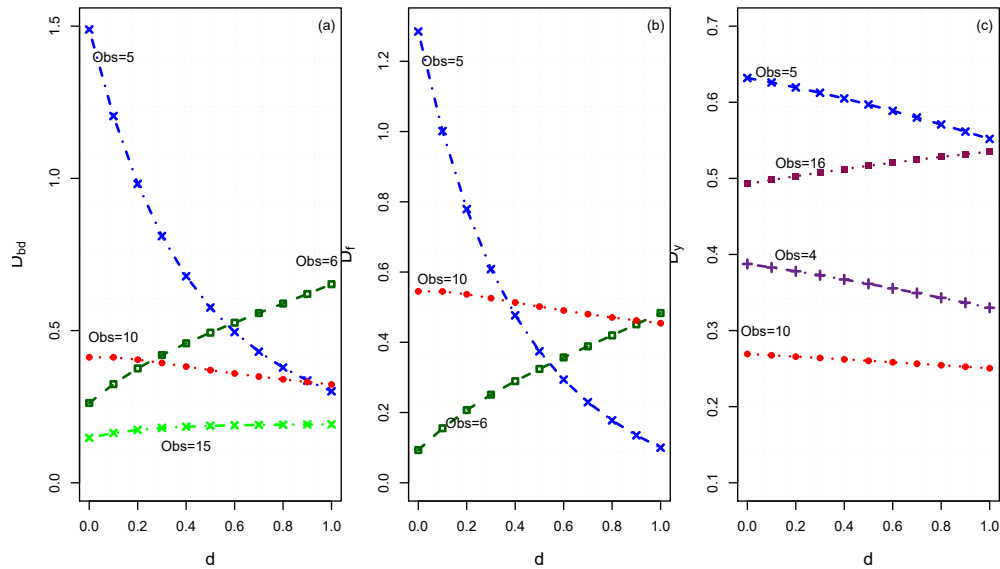


Figure 1: (a): Plot of D_{bi} against d , (b): Plot of D_{fi} against d , (c): Plot of D_{yi} against d

Table 1: The three most influential observations according to the D_{yi} , D_{bi} and D_{fi} in the simulation study

Σ	n	Obs.	D_{yi}	D_{bi}	D_{fi}
Σ_1	15	5	0.0943(0.0281)	0.1941(0.0105)	0.0605(0.1061)
		10	0.0328(0.0176)	0.0997(0.0755)	0.0710(0.1100)
		15	0.1003(0.0139)	0.1203(0.0182)	0.09114(0.0152)
		other	$\leq .02726(0.0301)$	$\leq .04174(0.0392)$	$\leq .0534(0.0608)$
	50	5	0.1253(0.0150)	0.1318(0.0194)	0.1967(0.0167)
		10	0.1827(0.0121)	0.101(0.0110)	0.0979(0.0182)
		15	0.228(0.0151)	0.2027(0.0141)	0.169(0.0315)
		other	$\leq .0493(0.0107)$	$\leq .0433(0.0100)$	$\leq .0735(0.0089)$
	100	5	0.5010(0.0081)	0.4010(0.0200)	0.2101(0.0092)
		10	0.4490(0.0087)	0.2783(0.0096)	0.2079(0.0099)
		15	0.3641(0.0078)	0.5502(0.0097)	0.3044(0.0105)
		other	$\leq .0140(0.0103)$	$\leq .0162(0.0091)$	$\leq .0094(0.0131)$
	Σ_2	15	5	0.129(0.0028)	0.491(0.0088)
10			0.3914(0.0044)	0.3152(0.0067)	0.2671(0.0061)
15			0.2149(0.0073)	0.3861(0.0092)	0.2918(0.0092)
other			$\leq .0621(0.0073)$	$\leq .0346(0.0290)$	$\leq .068(0.0053)$
50		5	0.3461(0.0055)	0.2183(0.0083)	0.2192(0.0099)
		10	0.3410(0.0098)	0.2093(0.0074)	0.1901(0.0039)
		15	0.2990(0.0048)	0.2088(0.0081)	0.1011(0.0102)
		other	$\leq .0194(0.0097)$	$\leq .0504(0.0082)$	$\leq .0089(0.0069)$
100		5	0.314(0.0064)	0.2100(0.0067)	0.1547(0.0066)
		10	0.2910(0.0083)	0.3094(0.0076)	0.2819(0.0086)
		15	0.2116(0.0076)	0.1512(0.0091)	0.1989(0.0043)
		other	$\leq .0069(0.0081)$	$\leq .0095(0.0056)$	$\leq .0080(0.0061)$
Σ_3		15	5	0.1319(0.0027)	0.2852(0.0089)
	10		0.2671(0.0082)	0.1845(0.0086)	0.1906(0.0089)
	15		0.2354(0.0028)	0.1839(0.0086)	0.251(0.0059)
	other		$\leq .0094(0.0079)$	$\leq .0154(0.0081)$	$\leq .063(0.0097)$
	50	5	0.1328(0.0080)	0.1715(0.0061)	0.3014(0.010)
		10	0.5951(0.0064)	0.2627(0.0074)	0.2199(0.0076)
		15	0.1498(0.0088)	0.2691(0.0026)	0.2990(0.0094)
		other	$\leq .0061(0.0084)$	$\leq .0069(0.0083)$	$\leq .0076(0.0099)$
	100	5	0.2891(0.0081)	0.2816(0.0099)	0.1107(0.0099)
		10	0.4446(0.0078)	0.2638(0.0066)	0.2091(0.0058)
		15	0.2951(0.0075)	0.2804(0.0029)	0.2769(0.0039)
		other	$\leq .0221(0.0049)$	$\leq .0086(0.0029)$	$\leq .0161(0.0097)$

Table 2: The percentage of correct detection in the simulation study with $n = 50$

Σ	% of outliers	D_{yi}	D_{bi}	D_{fi}
Σ_1	5%	100	100	99
	10%	99	98	97
	25%	98	99	97
Σ_2	5%	98	99	98
	10%	97	97	96
	25%	96	96	96
Σ_3	5%	97	97	95
	10%	96	96	94
	25%	95	96	94

Table 3: The 6 most influential observations: Longley data. Values with * indicates the cases with values larger than the reference values.

Case	d=0			d=0.962		
	D_{bi}	D_{fi}	D_{yi}	D_{bi}	D_{fi}	D_{yi}
4	0.005	0.005	0.378*	0.002	0.003	0.337*
5	1.489*	1.284*	0.631*	0.336*	0.135*	0.561*
6	0.262*	0.017	0.093	0.621*	0.451*	0.091
10	0.412*	0.545*	0.269*	0.331*	0.462*	0.250*
15	0.147*	0.190*	0.088	0.191*	0.191*	0.096
16	0.042	0.047	0.492*	0.054	0.059	0.532*

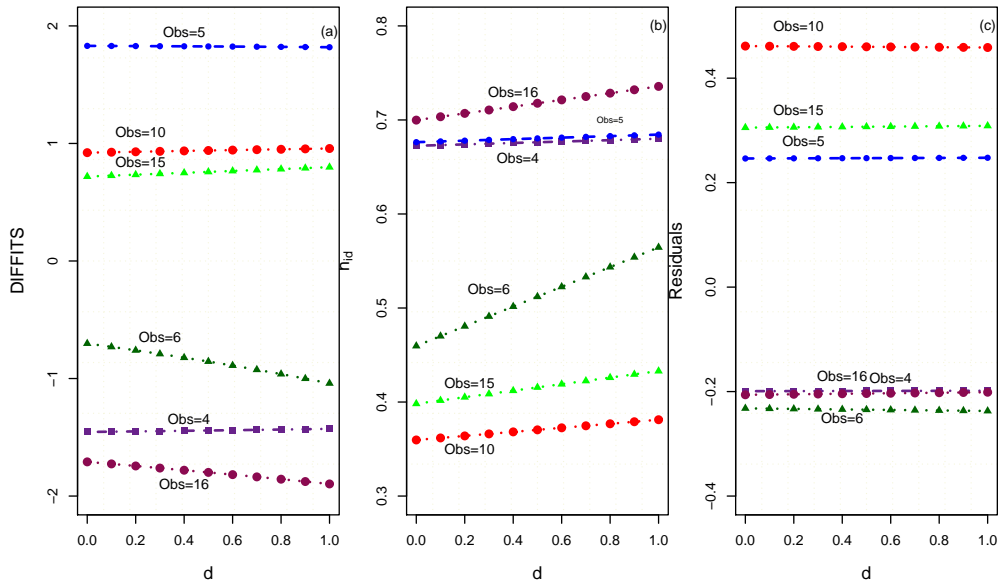


Figure 2: (a): Plot of DFFITS against d , (b): plot of leverages against d , (b): Plot of residuals against d

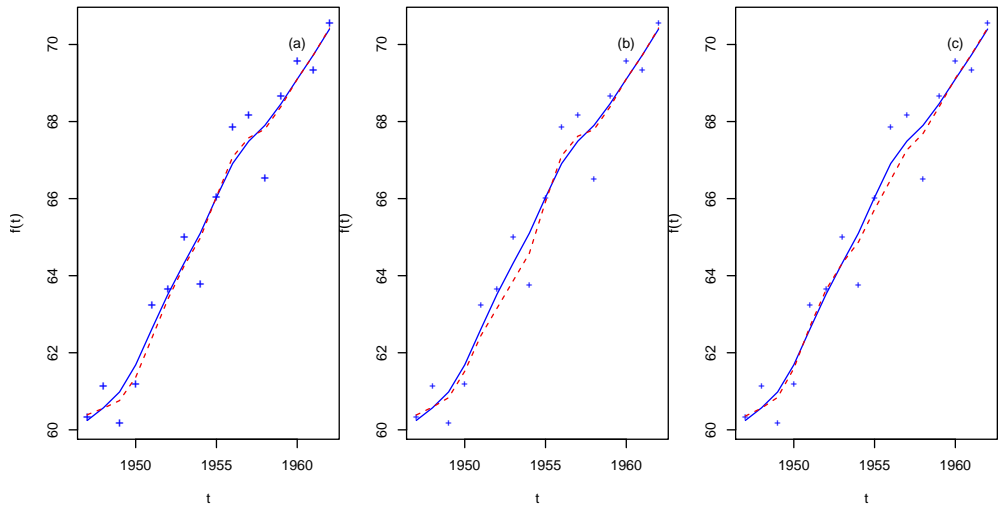


Figure 3: Plot of \hat{f}_d (solid line) when case 5 is deleted (dashed line) (a), when case 10 is deleted (b), when case 6 is deleted (c).

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